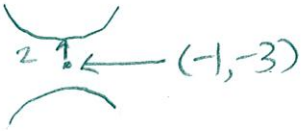
Find the co-ordinates of the vertices and foci, and the equations of the asymptotes of the hyperbola $x^2 - 9y^2 + 2x - 54y - 44 = 0$. **State clearly which co-ordinates are for which points.** **SCORE: ____ / 8 PTS**

$x^2 + 2x - 9y^2 - 54y = 44$

$(x^2 + 2x + 1) - 9(y^2 + 6y + 9) = 44 + 1 - 9 \cdot 9$

$(x+1)^2 - 9(y+3)^2 = -36$ (2)

$\frac{(y+3)^2}{4} - \frac{(x+1)^2}{36} = 1$ (1)



$c^2 = 4 + 36 = 40$

$c = 2\sqrt{10}$

CENTER $(-1, -3)$

VERTICES $(-1, -3 \pm 2) = (-1, -1), (-1, -5)$ (1 1/2)

FOCI $(-1, -3 \pm 2\sqrt{10})$ (1 1/2)

ASYMPTOTES SLOPE $= \pm \frac{\sqrt{4}}{\sqrt{36}} = \pm \frac{2}{6} = \pm \frac{1}{3}$ (1)

$y + 3 = \pm \frac{1}{3}(x + 1)$ (1)

Classify the graph of each equation as a circle, a parabola, an ellipse or a hyperbola. **SCORE: ____ / 2 PTS**

[a] $3x^2 + 5x + 4y + 1 = 0$ PARABOLA (1)

[b] $7x^2 + 7x + 5y^2 + 12y - 2 = 0$ ELLIPSE (1)

A point has polar coordinates $\left(14, \frac{4\pi}{9}\right)$. **SCORE: ____ / 2 PTS**

[a] Find another set of polar coordinates for the point, using a positive value of r.

$(14, \frac{4\pi}{9} \pm 2\pi) = (14, \frac{22\pi}{9})$ OR $(14, \frac{-14\pi}{9})$; (1) IF YOU GOT EITHER ANSWER

[b] Find another set of polar coordinates for the point, using a negative value of r.

$(-14, \frac{4\pi}{9} \pm \pi) = (-14, \frac{13\pi}{9})$ OR $(-14, \frac{-5\pi}{9})$; (1) IF YOU GOT EITHER ANSWER

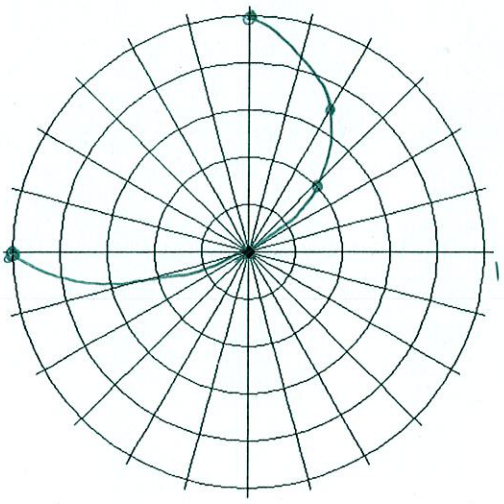
Plot the graph of $r = 2\sin\theta - 1$ for $0 \le \theta \le \frac{\pi}{2}$. You must list the polar coordinates for 5 points in that range. **SCORE: ____ / 8 PTS**

Use decimal approximations for irrational values of r.

POINTS ON GRAPH

(5)

- $(-1, 0)$
- $(0, \frac{\pi}{6})$
- $(0.4, \frac{\pi}{4})$
- $(0.7, \frac{\pi}{3})$
- $(1, \frac{\pi}{2})$



GRADED BY ME

Convert the rectangular coordinates $(-3\sqrt{3}, 9)$ to polar coordinates. **SCORE: ____ / 4 PTS**

$r = \sqrt{27+81} = \sqrt{108} = 6\sqrt{3}$

$\sin\theta = \frac{9}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{18} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{-3\sqrt{3}}{6\sqrt{3}} = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

(1) IF IN CORRECT ORDER

$(6\sqrt{3}, \frac{2\pi}{3})$

① ②

Convert the polar equation $r^2 = 3 + \sin 2\theta$ to rectangular form. **SCORE: ____ / 6 PTS**

$r^2 = 3 + 2\sin\theta\cos\theta$ (2)

$r^2 = 3 + \frac{2xy}{r^2}$ (1½)

$r^4 = 3r^2 + 2xy$ (1) ← CAN ALSO BE WRITTEN AS $r^2(r^2-3) = 2xy$

$(x^2+y^2)^2 = 3(x^2+y^2) + 2xy$

$(x^2+y^2)(x^2+y^2-3) = 2xy$ (1½)

IF YOU GOT EITHER ANSWER